which reduces to

$$(a_{ii} - a_{jj})_K = \sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}$$

Thus, after the annihilation of the off-diagonal element a_{ij}, $j > i$, the eigenvalues on the diagonal will be in descending order when $\sqrt{(a_{ii} - a_{jj})^2 + 4a_{ij}a_{ji}}$ is assigned a positive sign, and they will be in ascending order with a negative sign.

The sum of the squares of the elements in the rows that have been altered during the operation $X_K^T A_{K-1}$ remains unchanged. The proof is that for each column k of the affected rows i and j

$$\begin{aligned}(a_{ik}^2 + a_{jk}^2)_K &= (Sa_{ik} + Ca_{jk})_{K-1}^2 + (Ca_{ik} - Sa_{jk})_{K-1}^2 \\ &= S^2(a_{ik}^2 + a_{jk}^2) + C^2(a_{ik}^2 + a_{jk}^2) \\ &= (S^2 + C^2)(a_{ik}^2 + a_{jk}^2) \\ &= (a_{ik}^2 + a_{jk}^2)_{K-1}\end{aligned}$$

The preceding procedure can be repeated to show the invariance in the sum of the squares of the elements in the columns transformed from $X_K^T A_{K-1}$ to $X_K^T A_{K-1} X_K$.

It can be shown that the preceding discussion also applies to problems whose solutions are complex when the variable C in matrix X_K of Eq. (11), now a unitary matrix, is replaced by its complex counterpart above the diagonal, and its complex conjugate below the diagonal. But since the motion of algebraic order is nonexistent with complex numbers, arbitrating the sign of the quantity $(a_{ii} - a_{jj})_K$ via Eq. (18) serves no purpose. This lack of order seems to be the reason why some problems whose solution is complex will fail to converge—without deverging.

Numerical Examples

Figure 1 shows the details of the solution of a real problem with a real solution. Note that although the sign of the radical in Eq. (18) is set to produce eigenvalues of descending order on the diagonal of matrix Λ, the second and third eigenvalues do not comply to this expectation. This is because all above-diagonal elements had already been sufficiently annihilated just before these two values were to be processed.

Figure 2 shows the details of the solution of a real problem with a complex solution.

Conclusion

An eigenvalue problem solution using the Jacobi method was derived. In this method, the eigenvalues are inherently generated in algebraic order. This order is a necessary ingredient of the process since its absence will impede convergence of the unsymmetric problem solution. The notion of algebraic order is nonexistent with complex numbers. For that reason, this method will not always be convergent with problems whose solution is complex.

Acknowledgments

Dividing Eq. (16) by C^2 to arrive at Eq. (17), the prime mover behind this method, was the suggestion of J. Pickard. Also, thanks to C. G. Dietz, principal engineer at McDonnell Douglas Space Systems Company, Huntington Beach, CA, for the proof following Eq. (23).

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Generalized Proportional-Plus-Derivative Compensators for a Class of Uncertain Plants

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Introduction

THIS paper considers controller design for a class of systems consisting of a positive-real transfer function $P(s)$ followed by an integrator (Fig. 1). Examples of such plants include flexible space structures, which must be controlled with specified precision in attitude and shape. We assume that the order of $P(s)$, as well as the values of its parameters, are not accurately known; the only information available with certainty is that $P(s)$ is positive real, as defined below.[1]

Definition. A rational function $G(s)$ of the complex variable s is said to be positive real (PR) if 1) $G(s)$ is real when s is real, and 2) $\mathrm{Re}[G(s)] \geq 0$ for all $\mathrm{Re}[s] \geq 0$, where Re[.] denotes the real part.

It can be shown that PR functions have no zeros or poles in the open right half of the complex plane, and that the poles on the imaginary axis are simple (i.e., of multiplicity one), with non-negative residues.[1] It can be also shown that PR functions must have relative degree between -1 and 1 (relative degree is the difference between the degrees of the denominator and the numerator polynomials).

An example of the type of system shown in Fig. 1 arises in the study of large, flexible space structures. For example, the linearized single-axis rotational motion of a flexible space structure can be described by the transfer function:

$$G(s) = \frac{Y_a(s)}{U(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_q} \frac{\phi_i \psi_i}{s^2 + 2\rho_i\omega_i s + \omega_i^2} \tag{1}$$

where J is the moment of inertia, $y_a(t)$ denotes the total "attitude" at the sensor location, $u(t)$ is the applied control torque, ω_i, ρ_i, ϕ_i, and ψ_i are the elastic mode frequency, inherent damping ratio, and rotational component of the mode shape at the actuator and sensor locations, respectively, for the ith elastic mode. (ω_i are positive, and ρ_i are small positive numbers on the order of 0.01.) The number of modes included in the model is n_q. In theory, there are infinite modes; therefore, unmodeled dynamics are always present. If the actuator and sensor are collocated, then $\phi_i = \psi_i$. Obviously, $G(s)$ in Eq. (1) is not positive real because its relative degree is 2. However, consider the transfer function $P(s)$ between $u(t)$ and $y_r(t)$, where $y_r(t)$ is attitude rate $[=\dot{y}_a(t)]$:

$$P(s) = sG(s) = \frac{1}{Js} + \sum_{i=1}^{n_q} \frac{\phi_i^2 s}{s^2 + 2\rho_i\omega_i s + \omega_i^2} \tag{2}$$

It can be seen that $P(s)$ in Eq. (2) is indeed positive real. Furthermore, it is clear that $P(s)$ has only one pole on the imaginary axis ($s = 0$) and no zeros on the imaginary axis.

Definitions and Preliminaries

A system represented by the rational transfer function $H(s)$ (wherein the numerator and denominator polynomials are

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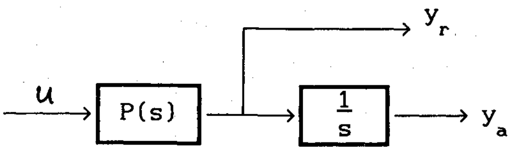

Fig. 1 Plant under consideration.

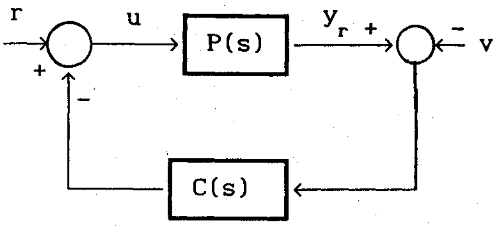

Fig. 2 Feedback configuration.

coprime) is defined to be *stable* if all the poles of $H(s)$ are in the open left-half plane (OLHP), i.e., its denominator polynomial is Hurwitz. $H(s)$ is said to be "minimum phase" if its numerator polynomial is Hurwitz. A real square matrix is said to be Hurwitz if its characteristic polynomial is Hurwitz. Figure 2 shows a feedback configuration wherein $P(s)$ and $C(s)$ denote the plant and compensator transfer functions. The input v represents a disturbance (e.g., sensor noise) and must be included for stability analysis in the s domain.[2] The controller C is said to stabilize a plant P if all of the following transfer functions are stable[2]:

1) Closed-loop transfer function from r to y_r: $G_{CL} = P/(1 + PC)$.

2) Transfer function from r to u: $G_1 = 1/(1 + PC)$.

3) Transfer function from v to u: $G_2 = C/(1 + PC)$.

It is necessary to check the stability of all three of the transfer functions to ensure that all signals in the closed-loop system remain bounded in the presence of bounded inputs. To clarify this, let $P(s) = N_p(s)/D_p(s)$ and $C(s) = N_c(s)/D_c(s)$, where N and D denote the numerator and denominator polynomials for the subscript system. For the sake of generality, we assume that the respective N and D are not necessarily coprime, i.e., they may have common divisors (i.e., common zeros). Denoting $\beta(s) = D_p(s)D_c(s) + N_p(s)N_c(s)$, we have

$$G_{CL} = \frac{N_p D_c}{\beta}, \qquad G_1 = \frac{D_p D_c}{\beta}, \qquad G_2 = \frac{N_c D_p}{\beta} \tag{3}$$

Because of the possibility of cancellations of different common factors in the three transfer functions in Eq. (3), the stability of G_{CL} does not generally imply that of the other two. From Ref. 2, if $C(s)$ is stable, the stability of G_{CL} implies that of G_1 and G_2. We now present a weaker condition. First we prove the following:

Fact 1. Suppose each of the functions $P(s)$ and $C(s)$ has no unstable cancellations [i.e., no common poles and zeros in the closed right-half plane (CRHP)]. Then

1) $(s - \sigma)$ [with Re[σ] $\geq$ 0] is a common divisor of $N_p(s)D_c(s)$ and $\beta(s)$ iff it is a common divisor of $N_p(s)$ and $D_c(s)$.

2) $(s - \sigma)$ [with Re[σ] $\geq$ 0] is a common divisor of $D_p(s)D_c(s)$ and $\beta(s)$ iff it is a common divisor of $N_p(s)$ and $D_c(s)$ or of $N_c(s)$ and $D_p(s)$.

3) $(s - \sigma)$ [with Re[σ] $\geq$ 0] is a common divisor of $N_c(s)D_p(s)$ and $\beta(s)$ iff it is a common divisor of $N_c(s)$ and $D_p(s)$.

Proof. The "sufficiency" in element 1 is obvious. To prove the "necessity," suppose $(s - \sigma)$ is a divisor of N_pD_c and β. If $(s - \sigma)$ is a divisor of N_p and β, then $N_p(\sigma) = \beta(\sigma) = 0$, i.e., $(s - \sigma)$ must be a divisor of D_p or D_c. Since (N_p,D_p) has no unstable common divisors, $(s - \sigma)$ must be a divisor of D_c. Similarly, if $(s - \sigma)$ divides D_c and β, it also divides N_p. Elements 2 and 3 can be proved in a similar manner.

From fact 1, it can be concluded that if there are no unstable cancellations in the product $P(s)C(s)$, then there are no unstable cancellations in G_{CL}, G_1 and G_2. In other words, we have the following:

Fact 2. If $P(s)C(s)$ has no unstable cancellations, the stability of any one of the transfer functions, $G_{CL}(s)$, $G_1(s)$, or $G_2(s)$ implies that of the other two.

For the present problem, to prove the stability, we shall assume that PC has no unstable cancellations. The stability of $G_1(s)$ would then be sufficient to prove the stability of the system. If G_1 is stable and if $P(s)$ and $C(s)$ are represented by stabilizable and detectable state-space realizations, the resulting state-space model for the closed-loop system will be asymptotically stable,[2] i.e., the input/output stability and asymptotic stability are equivalent. [Note that the additional requirement for being well posed,[2] i.e., $1 + P(\infty)C(\infty) \neq 0$, is always satisfied when P and C are PR, since they have no poles or zeros in the open right-half plane (ORHP)].

The problem considered herein is to find a feedback controller $C(s)$, which uses the feedback of the output y_a and the output rate y_r, to robustly stabilize the plant, i.e., to stabilize the plant despite uncertainties in the plant order and the plant parameters. We first consider the related problem of stabilization of the plant $P(s)$ only, with the feedback of output rate.

Output-Rate Stabilization Using Positive-Real Controllers

In this section, we investigate the stabilization of the system $P(s)$ only, with no integrator at the output. In the case of large space structures, this implies the stabilization of the attitude rate (i.e., with zero external inputs, the attitude rate should tend to zero as $t \to \infty$). We first prove the following lemma. [It should be noted that similar results have appeared in the literature in the past (for the case with no integrator at the output), but they use the concept of strictly positive real (SPR) functions, which is more restrictive. Furthermore, there appear to be several nonequivalent definitions of SPR systems, leading to ambiguity. For these reasons, we do not use the SPR concept in this paper.]

Lemma 1. For the block diagram shown in Fig. 2, suppose both $P(s)$ and $C(s)$ are PR. Then the closed-loop system $G_{CL} = P/(1 + PC)$ is PR. Furthermore, if there are no unstable cancellations in $P(s)C(s)$, and if *either* Re[$P(j\omega)$]>0 *or* Re[$C(j\omega)$]>0 *or both* >0, for all real ω, then $C(s)$ stabilizes $P(s)$.

Proof. The proof consists of three steps: 1) Prove that $\beta(s)$ has no ORHP zeros, i.e., G_1 has no ORHP poles; 2) G_1 has no poles on the imaginary axis, i.e., G_1 is stable; and 3) C stabilizes P. Let

$$P(j\omega) = \mathrm{R}_P(\omega) + j\mathrm{I}_P(\omega), \qquad C(\omega) = \mathrm{R}_C(\omega) + j\mathrm{I}_C(\omega) \tag{4}$$

where R and I denote the real and the imaginary part of the subscript function. Then

$$G_{CL}(j\omega) = \frac{P}{1 + PC} = \frac{P(1 + \bar{P}\bar{C})}{(1 + PC)(1 + \bar{P}\bar{C})} = \frac{P + |P|^2\bar{C}}{|1 + PC|^2} \tag{5}$$

where the overhead bar denotes the complex conjugate. Therefore,

$$\mathrm{Re}[G_{CL}(j\omega)] = [\mathrm{R}_P + |P|^2\mathrm{R}_C]/\Delta \tag{6}$$

where Δ denotes the denominator in Eq. (5). Thus Re[$G_{CL}(\omega)$] ≥ 0, and G_{CL} is PR.

Since G_{CL} is PR, it has no poles in the ORHP. Since PC has no unstable cancellations, from the first element of fact 1, β

has no zeros in the ORHP. Denote

$$G_1(s) = \frac{1}{1 + P(s)C(s)} = \frac{D_p(s)D_c(s)}{\beta(s)} = \frac{N_1(s)}{D_1(s)} \tag{7}$$

where N_1 and D_1 are the numerator and denominator polynomials obtained after cancellations, if any. Then $D_1(s)$ has no ORHP zeros. Now we shall show that $D_1(s)$ has no zeros on the imaginary axis. To accomplish this, we prove that the equation:

$$1 + P(j\omega)C(j\omega) = 0 \tag{8}$$

cannot hold for any real ω. Considering the real and imaginary parts separately, Eq. (8) yields:

$$1 + \mathrm{R}_P\mathrm{R}_C - \mathrm{I}_P\mathrm{I}_C = 0 \tag{9}$$

$$\mathrm{I}_P\mathrm{R}_C + \mathrm{R}_P\mathrm{I}_C = 0 \tag{10}$$

Multiplying Eq. (9) by R_P and Eq. (10) by I_P, and adding

$$\mathrm{R}_P + |P|^2\mathrm{R}_C = 0 \tag{11}$$

Since P and C are entirely interchangeable, we have:

$$\mathrm{R}_C + |C|^2\mathrm{R}_P = 0 \tag{12}$$

If $\mathrm{R}_P > 0$, Eq. (11) cannot hold; and if $\mathrm{R}_C > 0$, Eq. (12) cannot hold. Therefore, Eq. (8) cannot be true for any real ω, and $D_1(s)$ is Hurwitz, i.e., G_1 is stable. Using fact 2, since $P(s)C(s)$ has no unstable cancellations, G_2 and G_{CL} are also stable, and $C(s)$ stabilizes $P(s)$.

The attractive feature of the preceding result is that the closed-loop stability is guaranteed regardless of uncertainties in the model order as well as in the parameters. However, the main problem is the stabilization of the output y_a, rather than the output rate y_r. We address this problem in the next section.

Generalized PD-Type Compensators

For the system shown in Fig. 2, we consider a class of control laws represented by

$$U(s) = R(s) - [G_p(s)Y_a(s) + G_r(s)Y_r(s)] \tag{13}$$

where $G_p(s)$ and $G_r(s)$ are proper rational transfer functions representing the generalized position and rate gains (Fig. 3). In the case of large flexible space structures, it is well known that the control law [Eq. (13)] with any positive constants G_p and G_r will stabilize the plant regardless of the model order or the parameter uncertainties.[3] Our objective herein is to generalize this result to the case where $P(s)$ is any PR plant and $G_p(s)$ and $G_r(s)$ are linear dynamic systems.

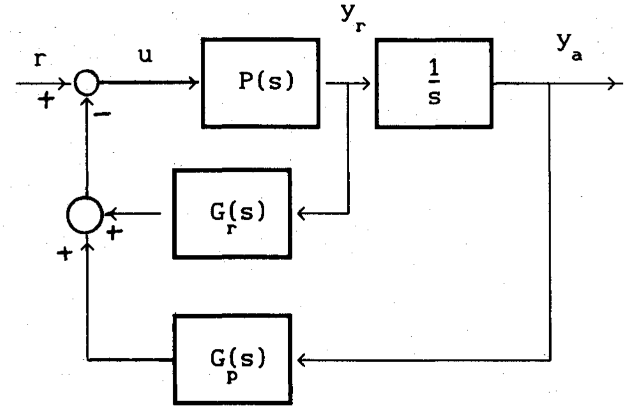

Fig. 3 Generalized *P-D* compensator.

The closed-loop system under consideration can be represented by the block diagram shown in Fig. 3. Our aim is to investigate the asymptotic stability of the closed-loop system. Let (A_p, B_p, C_p, E_p), $(A_{Gp}, B_{Gp}, C_{Gp}, E_{Gp})$, and $(A_{Gr}, B_{Gr}, C_{Gr}, E_{Gr})$ be minimal realizations of $P(s)$, $G_p(s)$, and $G_r(s)$, where A, B, C, and E, respectively, denote the system matrix, the input matrix, the output matrix, and the direct transmission matrix for the subscript system. Denoting the state vectors of P, G_p, and G_r by x_p, x_{Gp}, and x_{Gr}, respectively, the homogeneous part of the closed-loop system is then given by:

$$\dot{x} = A_{CL}x \tag{14}$$

where

$$x = (x_p^T, x_c^T)^T, \qquad x_c = (x_{Gp}^T, y_a, x_{Gr}^T)^T,$$

$$A_{CL} = \begin{bmatrix} A_p + B_pQ_p & B_pQ_c \\ B_{c1} + B_{c2}Q_p & A_c + B_{c2}Q_c \end{bmatrix} \tag{15}$$

$$Q_p = -(1 + E_{Gr}E_p)^{-1}E_{Gr}C_p, \quad Q_c = -(1 + E_{Gr}E_p)^{-1}C_c$$

$$B_{c1} = [0\ C_p^T\ (B_{Gr}C_p)^T]^T, \qquad B_{c2} = [0\ E_p^T\ (B_{Gr}E_p)^T]^T,$$

$$C_c = [C_{Gp}\ E_{Gp}\ C_{Gr}]$$

(Note that the E matrix for $C(s)$ is E_{Gr}, and that $E_{Gr}E_p \geq 0$ because P and C are PR. The condition for being "well-posed," i.e., $E_{Gr}E_p \neq -1$, is therefore satisfied.) The following theorem gives a generalized proportional plus derivative (PD)-type compensator that results in closed-loop asymptotic stability.

Theorem 1. The closed-loop system matrix A_{CL} in Eq. (15) is Hurwitz if the following (sufficient) conditions are satisfied:

1) $P(s)$ is PR.

2) $C(s) = \dfrac{G_p(s) + sG_r(s)}{s}$ is PR.

3) $P(s)C(s)$ has no unstable cancellations.

4) *Either* $\mathrm{Re}[P(j\omega)] > 0$, *or* $\mathrm{Re}[C(j\omega)] > 0$ *or both* > 0 for all real ω.

Proof. From lemma 1, $C(s)$ stabilizes $P(s)$; equivalently, if $P(s)$ and $C(s)$ are represented by stabilizable and detectable realizations, the composite closed-loop system is asymptotically stable. Let n_c denote the degree of the denominator of $C(s)$. Since $C(s)$ has no unstable cancellations, every n_c order realization of $C(s)$ is stabilizable and detectable. In particular, the following realization is stabilizable and detectable:

$$\dot{x}_{Gp} = A_{Gp}x_{Gp} + B_{Gp}y_a$$

$$\dot{y}_a = y_r$$

$$\dot{x}_{Gr} = A_{Gr}x_{Gr} + B_{Gr}y_r$$

$$y_c = C_{Gp}x_{Gp} + C_{Gr}x_{Gr} + E_{Gp}y_a + E_{Gr}y_r$$

where y_c denotes the compensator output. Combining the preceding equations with the plant state equation, the closed-loop matrix can be shown to be A_{CL}. Therefore, A_{CL} is Hurwitz.

Theorem 1 gives a generalized proportional-plus-derivative compensator that robustly stabilizes the plant. The following corollary is an immediate consequence of Theorem 1.

Corollary 1. The matrix A_{CL} in Eq. (15) is Hurwitz if all of the following conditions are satisfied:

1) $P(s)$ is PR and has no zeros at the origin.

2) $G_p(s)$, $G_r(s)$ are stable, G_p has no zeros at the origin, and $G_r(s)$, $[G_p(s)/s]$ are PR.

3) $\text{Re}[G_r(j\omega)] > 0$ for all real ω.

Proof. From assumption 2, $C(s)$ is PR and has no unstable cancellations. From assumptions 1 and 2, $P(s)C(s)$ has no unstable cancellations. From assumption 3, $\text{Re}[C(j\omega)] > 0$ for all real ω. Therefore, from Theorem 1, A_{CL} is Hurwitz.

A special case of particular interest is the flexible spacecraft control problem, where P is given by Eq. (2) and G_p, G_r are positive constants, and a finite-bandwidth actuator, represented by a stable, proper, minimum-phase transfer function $G_a(s)$, is used in the loop. For this case,

$$C(s) = \frac{G_a(s)[G_p + sG_r]}{s} \tag{16}$$

From Eq. (2), $P(s)$ is PR and has no zeros at $s = 0$. Therfore, from Theorem 1, A_{CL} is stable if $\text{Re}[C(j\omega)] > 0$ for all real ω, which will be satisfied if

$$-90 \text{ deg} < \phi[C(j\omega)] < 90 \text{ deg} \qquad \text{for } 0 \leq \omega < \infty$$

where $\phi[.]$ denotes the phase angle of a complex variable. The preceding expression can be shown to be equivalent to

$$-\arctan(\omega G_r/G_p) < \phi[G_a(j\omega)] < 180 \text{ deg} - \arctan(\omega G_r/G_p)$$
$$\text{for } 0 \leq \omega < \infty \tag{17}$$

For the special case where $G_a(s) = k/(s + a)$(with $k > 0$, $a > 0$), Eq. (17) is equivalent to

$$a > G_p/G_r$$

That is, if the actuator bandwidth is greater than G_p/G_r, the closed-loop stability is guaranteed regardless of the number of modes in the model or parameter uncertainties. This result was proved in Ref. 4 for the multi-input/multi-output case for systems with constant, positive definite proportional and rate gain matrices, using function space methods.

Conclusions

A generalized proportional-plus-derivative compensator was proposed for robustly stabilizing a class of uncertain plants represented by a positive-real transfer function followed by an integrator. Such plants are encountered, for example, in the study of attitude control of large flexible space structures, wherein significant uncertainty exists in the model order as well as the parameter values. The novel feature of the proposed compensator is that the proportional and rate gains used are transfer functions rather than constants, which allows more design freedom, and offers the potential for obtaining better performance with guaranteed robust stability. The results obtained herein are for single-input/single-output systems. It would be highly desirable to extend the results to the multivariable case, which will then be applicable to realistic flexible spacecraft, and also to develop synthesis methods for such controllers.

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Order-Variable Adaptive Pole-Placement Controllers for a Flexible System

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I. Introduction

ORDER determination is an important problem in adaptive control of flexible systems because the number of plant modes excited varies with external excitation, speed of maneuvers, etc. Also, the number of excited modes can increase abruptly due to impulsive disturbances such as collisions or release of payload. The number of additional modes and their contribution to output often are larger than levels accommodated by robust controllers of lower order. In such applications, a major difficulty is to identify the correct order of the system (and adjust the control law). Here, we propose a natural approach to the issue of unknown, or varying, system order; change the order of the control law, as the need arises. Such a technique requires the ability to identify adaptively the appropriate order for the system while providing the parameter estimates corresponding to that order. Since this order is in general unknown, common identification algorithms, which have a fixed order, are not suitable for this purpose. The identification method used here is the lattice filter, which is an order-recursive method. Because of this order-recursive property, lattices provide the ability to obtain parameter estimates for any order model. The example in this paper involves a linear lumped-mass plant that increases in dimension after a collision during operation. Before the collision, the adaptive controller identifies a plant order and a corresponding set of parameters; after the collision, the controller must adjust both the order and the parameters. Simulations for the example show that the variable-order adaptive controller is much better than fixed-order controllers using the same control law. The adaptive control law used in this paper is an indirect pole-placement algorithm. Results presented show the advantages gained by allowing the order to change when the effective order of the system is either unknown or can change during operation.

II. Adaptive Control Law

For the example in Sec. IV indirect pole placement is chosen since it does not require the system to be minimum phase (e.g., see Ref. 1 for details). An auto regressive moving average (ARMA) model of the plant is identified adaptively, and the control gains are computed so that the closed-loop systems will have specified poles. Consider the input/output model

$$A(q^{-1})y(k) = B(q^{-1})u(k) \tag{1}$$

where $A(q^{-1}) = 1 - a_1q^{-1} - \cdots - a_nq^{-n}$, and q^{-1} is the backward shift; i.e., $q^{-1}y(k) = y(k-1)$. The control has the following form (see Ref. 2 for details of the pole-placement algorithm):

$$L(q^{-1})u(k) = P(q^{-1})[y^*(k+1) - y(k)] \tag{2}$$

where $L(q^{-1})$ and $P(q^{-1})$ are $(n-1)$ order polynomials in q^{-1}. Using Eq. (2) in Eq. (1) results in the following equation

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